# Continuous-Phase Frequency-Shift Keying (CPFSK) & Continuous-Phase Modulation (CPM)

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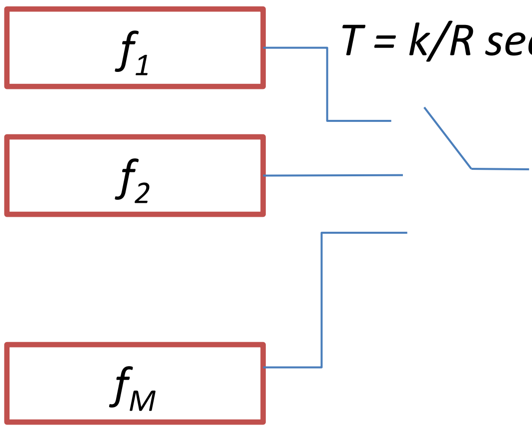
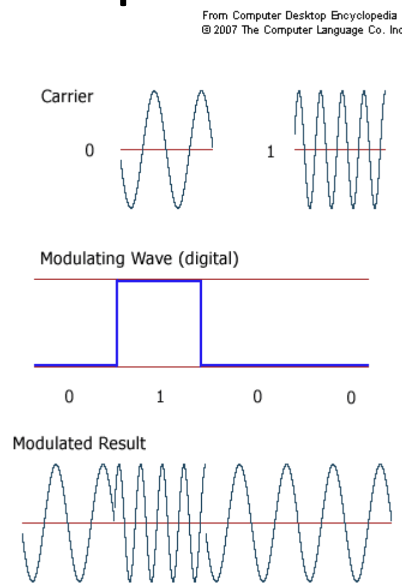
# Continuous Phase Frequency Shift Keying (CPFSK)

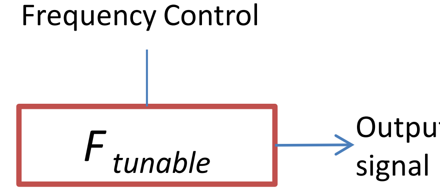
- CPM :the phase of the signal is constrained to be continuous ➔ Memory.
- For FSK, there are two options




*T = k/R seconds.*

$m\,\Delta f,\ 1 \le m \le M,\ \ M=2^k$




The information-bearing signal frequency modulates a single carrier whose frequency is changed continuously. The resulting frequency modulated signal is phase-continuous, (CPFSK)

Large spectral side lobes outside of the main spectral band of the signal (bandwidth requirements)

# Representation of CPFSK

- $\{a_n\}$ information sequence 0010101111101
- $\{I_n\}$ *sequence of amplitudes obtained by mapping k-bit blocks of* binary digits from the information sequence $\{a_n\}$ *into the amplitude levels ±1,±3, . . . , ±(M − 1).*
- $g(t)$ pulse shape. Example: rectangular pulse of amplitude *1/2T and duration T seconds.*
- $d(t)$ *is used to frequency-modulate the carrier*

$$d(t) = \sum_n I_n g(t - nT)$$

- $v(t)$ the equivalent lowpass waveform
- $f_d$ *is the* ***peak frequency deviation*** *and*
- $\varphi_0$ *is the initial phase of the carrier*

$$v(t) = \sqrt{\frac{2\mathcal{E}}{T}}\, e^{j\left[4\pi T f_d \int_{-\infty}^{t} d(\tau)\,d\tau + \phi_0\right]}$$

- The carrier-modulated signal corresponding

$$s(t) = \sqrt{\frac{2\mathcal{E}}{T}} \cos\left[2\pi f_c t + \phi(t; \boldsymbol{I}) + \phi_0\right]$$

- *φ(t; I) represents the time-varying phase of the carrier*

$$\begin{aligned}\phi(t; \boldsymbol{I}) &= 4\pi T f_d \int_{-\infty}^{t} d(\tau)\,d\tau \\ &= 4\pi T f_d \int_{-\infty}^{t} \left[\sum_n I_n g(\tau - nT)\right] d\tau\end{aligned}$$

# CPFSK

- Although *d(t) contains discontinuities, the integral of d(t) is continuous.*
- The phase of the carrier in the interval $nT \le t \le (n+1)T$

$$\phi(t;I) = 4\pi f_d T \int_{-\infty}^{t} d(\tau)d\tau = 4\pi f_d T \int_{-\infty}^{t} \left[ \sum_n I_n g(\tau - nT) \right] d\tau$$

$$\phi(t;I) = 4\pi f_d T \int_{-\infty}^{t} \left[ \sum_{k=-\infty}^{n-1} I_k g(\tau - kT) \right] d\tau + 4\pi f_d T \int_{-\infty}^{t} I_n g(\tau - nT) d\tau$$

Note that the $g(t)$ is assumed to be rectangular pulse of amplitude $1/2T$ and duration $T$ seconds.

$$\phi(t;I) = 2\pi f_d T \int_{-\infty}^{t} \left[ \sum_{k=-\infty}^{n-1} I_k \right] d\tau + 2\pi f_d T q(\tau - nT) I_n = \theta_n + 2\pi h I_n q(\tau - nT)$$

*$h=2f_dT$ ,The parameter h is called the **modulation index**.*

$$\theta_n = \pi h \sum_{k=-\infty}^{n-1} I_k$$

We observe that $\theta_n$ *represents the accumulation (memory) of all symbols up to time (n − 1)T .*

$$q(t) = \begin{cases} 0 & t < 0 \\ \frac{t}{2T} & 0 \le t \le T \\ \frac{1}{2} & t > T \end{cases}$$

# Continuous-Phase Modulation (CPM)

- CPFSK is a special case of CPM

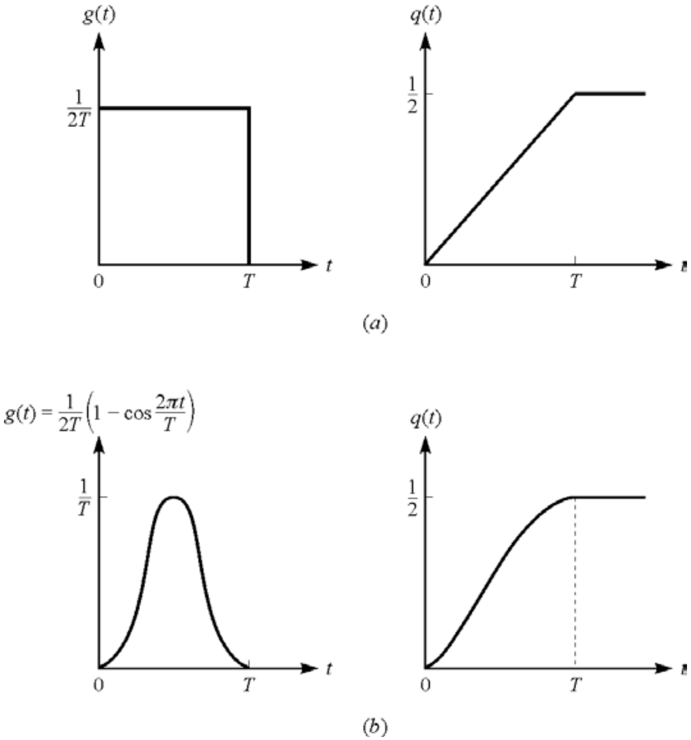
$$\phi(t; \boldsymbol{I}) = 2\pi \sum_{k=-\infty}^{n} I_k h_k q(t - kT), \qquad nT \le t \le (n+1)T$$

When $h_k = h$ *for all k, the modulation index is fixed* for all symbols. When the modulation index varies from one symbol to another, the signal is called **multi-*h* CPM.**

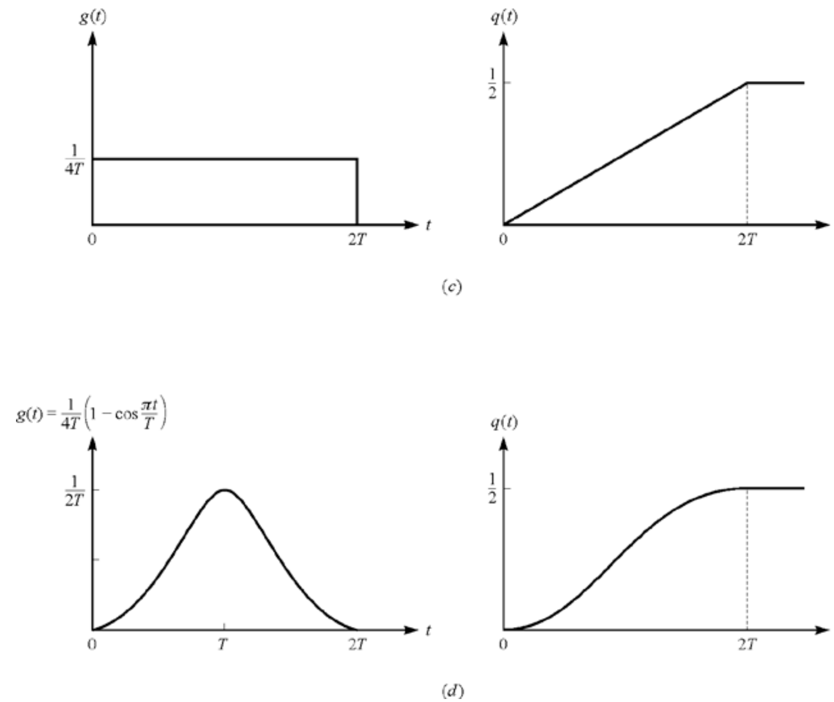
If $g(t) = 0$ *for* $t > T$ *, the signal is called* ***full-response CPM***. *If* $g(t) >< 0$ *for* $t > T$ *, the* modulated signal is called ***partial-response CPM.***

$$q(t) = \int_0^t g(\tau)\, d\tau$$

# Pulse Shapes for CPM




Pulse shapes for full response CPM




Pulse shapes for partial response CPM.

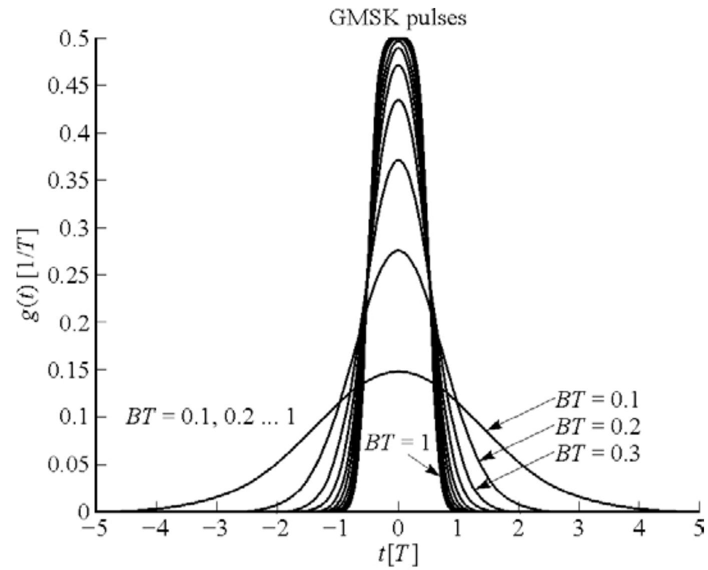
REC=Rectangle
RC=Raised Cosine
For *L > 1, additional memory is introduced in the CPM signal by the pulse g(t)*

**Some Commonly Used CPM Pulse Shapes**

| | |
|---|---|
| LREC | $g(t) = \begin{cases} \frac{1}{2LT} & 0 \le t \le LT \\ 0 & \text{otherwise} \end{cases}$ |
| LRC | $g(t) = \begin{cases} \frac{1}{2LT}\left(1 - \cos\frac{2\pi t}{LT}\right) & 0 \le t \le LT \\ 0 & \text{otherwise} \end{cases}$ |
| GMSK | $g(t) = \frac{Q\left(2\pi B\left(t - \frac{T}{2}\right)\right) - Q\left(2\pi B\left(t + \frac{T}{2}\right)\right)}{\sqrt{\ln 2}}$ |

# *Gaussian minimum-shift keying* (GMSK)

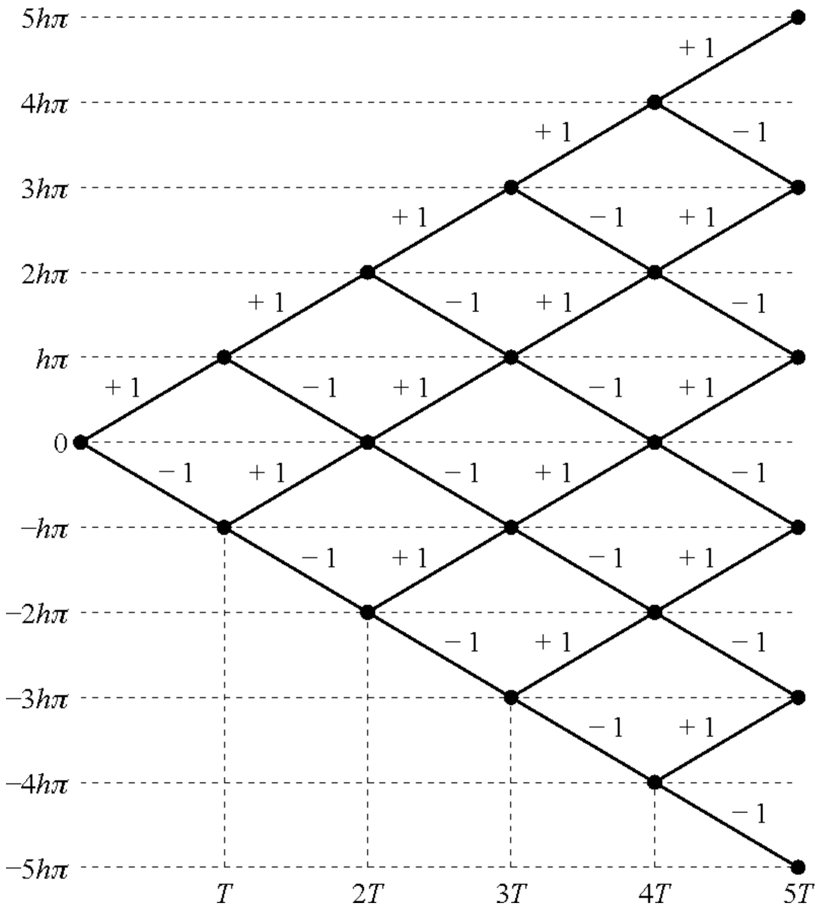
- GMSK with *BT = 0.3* is used in the European digital cellular communication system, called **GSM**. We observe that when *BT = 0.3, the GMSK pulse may be truncated* at |*t*| = *1.5T with a relatively small error incurred for t > 1.5T*



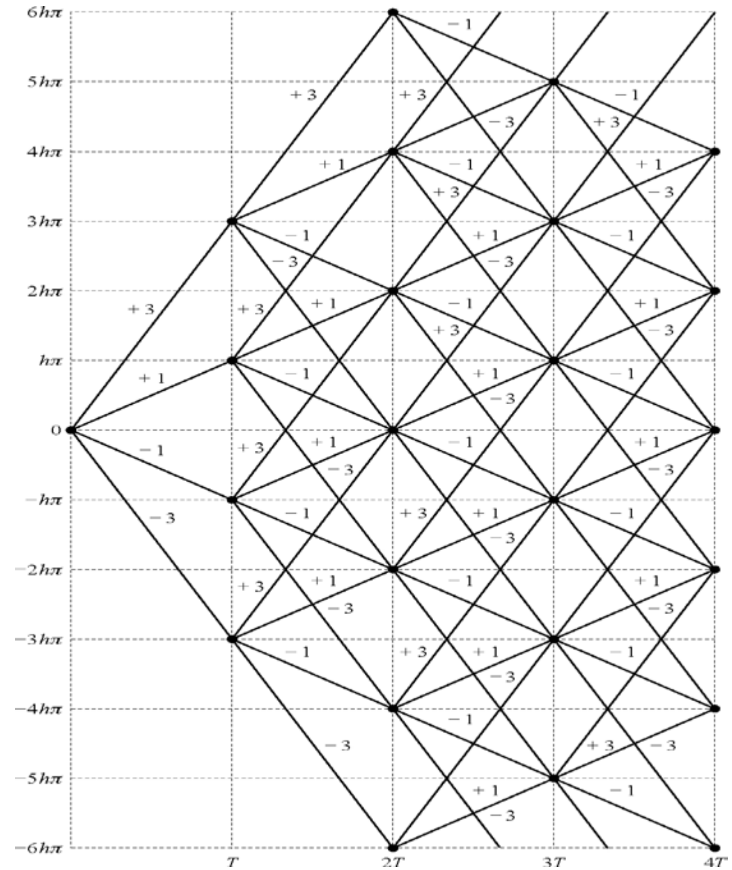

(e)

*B, which represents the −3-dB bandwidth* of the Gaussian pulse

# Phase trajectory for CPFSK.

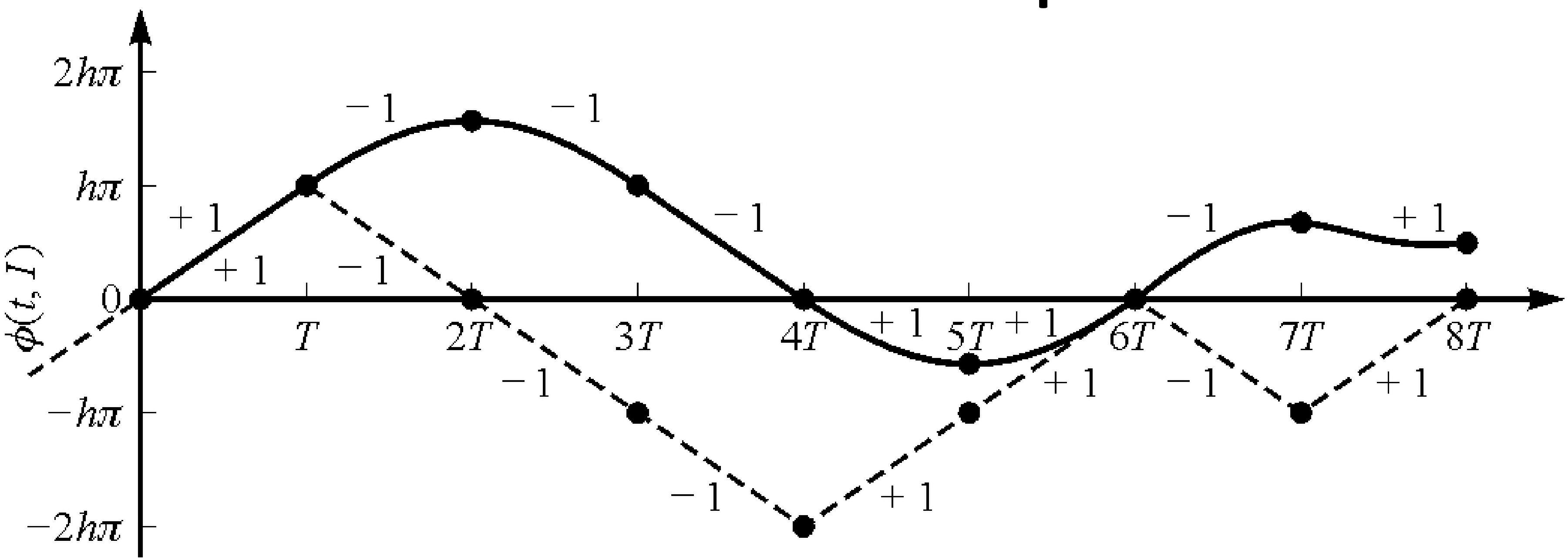



Phase trajectory for binary CPFSK



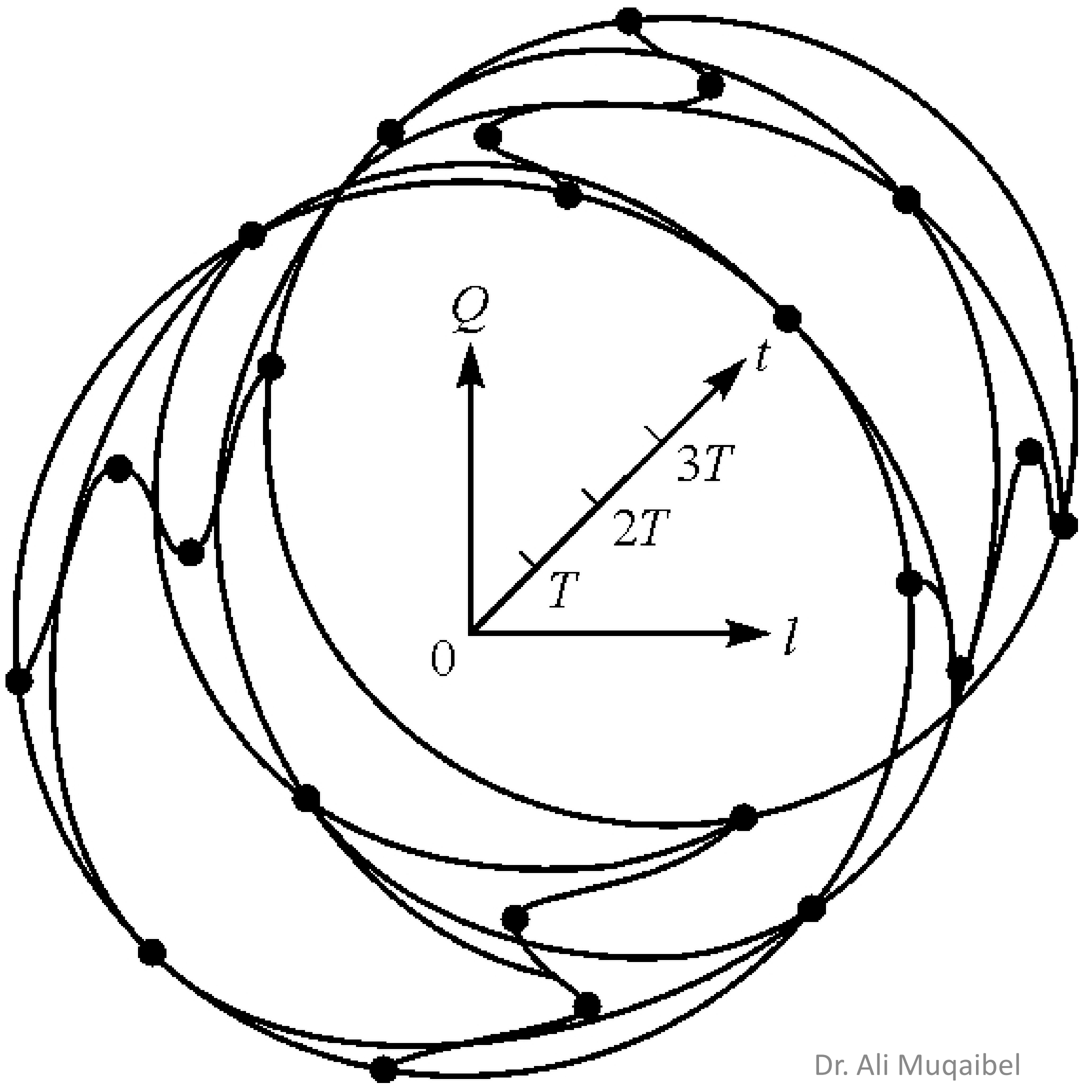

Phase trajectory for quaternary CPFSK

# Piecewise vs. smooth phase tree



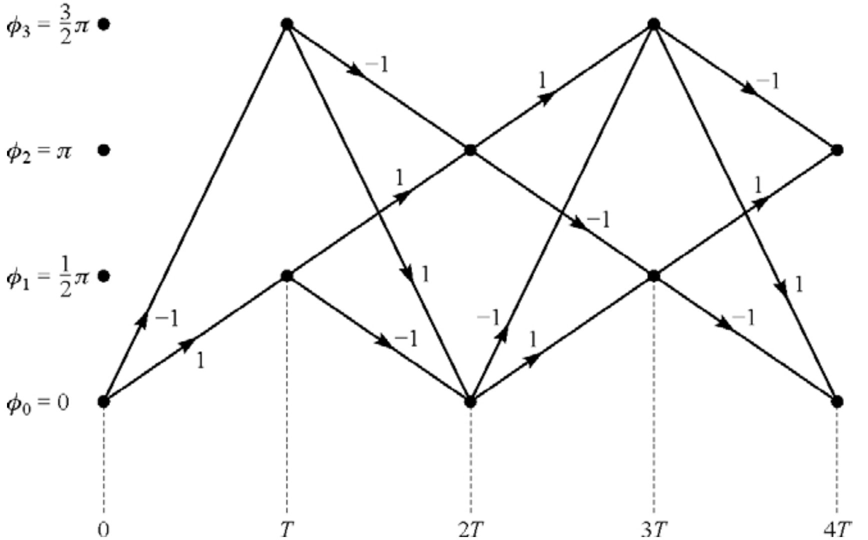

Phase trajectories for binary CPFSK (dashed) and binary, partial response CPM based on raised cosine pulse of length 3*T* (solid). [*From Sundberg (1986), © 1986 IEEE.*]

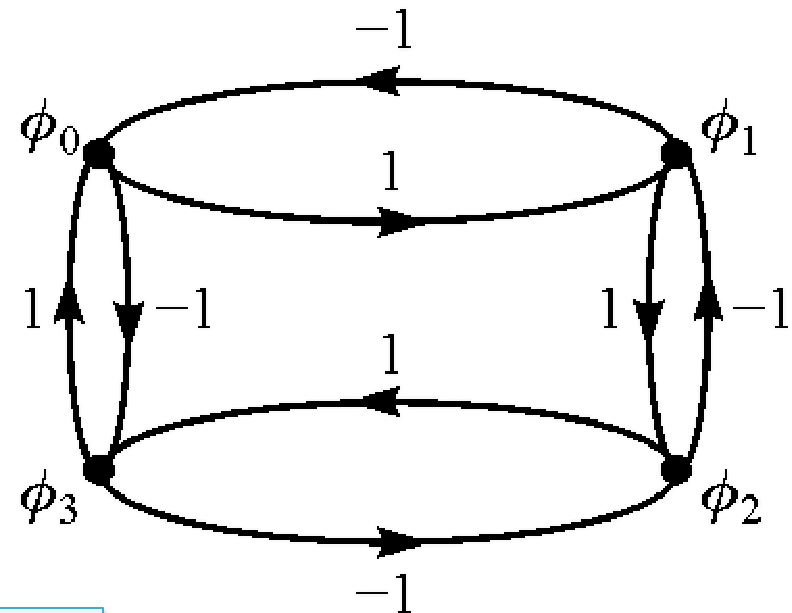
# Phase cylinder




Phase cylinder for binary CPM with *h* = ½ and a raised cosine pulse of length 3*T*. [*From Sundberg (1986), © 1986 IEEE.*

# State Trellis & State diagram




The phase transitions from one state to another are not true phase trajectories. They represent phase transitions for the (terminal) states at the time instants $t = nT$ .

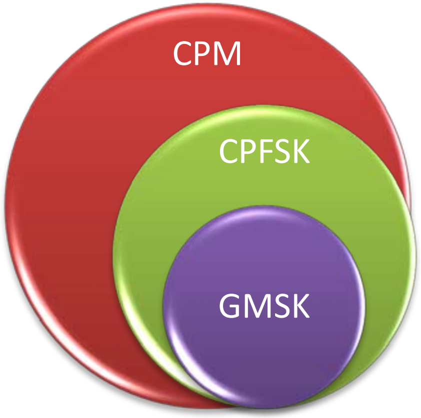
State trellis for binary CPFSK with $h$ = ½

# Minimum Shift Keying (MSK)

MSK is a special form of binary CPFSK (and, therefore, CPM) in which the modulation index *h = ½* and *g(t) is a rectangular pulse of duration T*

$$\phi(t; \boldsymbol{I}) = \frac{1}{2}\pi \sum_{k=-\infty}^{n-1} I_k + \pi I_n q(t - nT)$$

$$= \theta_n + \frac{1}{2}\pi I_n \left(\frac{t - nT}{T}\right), \qquad nT \le t \le (n+1)T$$




$$s(t) = A \cos\left[2\pi f_c t + \theta_n + \frac{1}{2}\pi I_n \left(\frac{t - nT}{T}\right)\right]$$

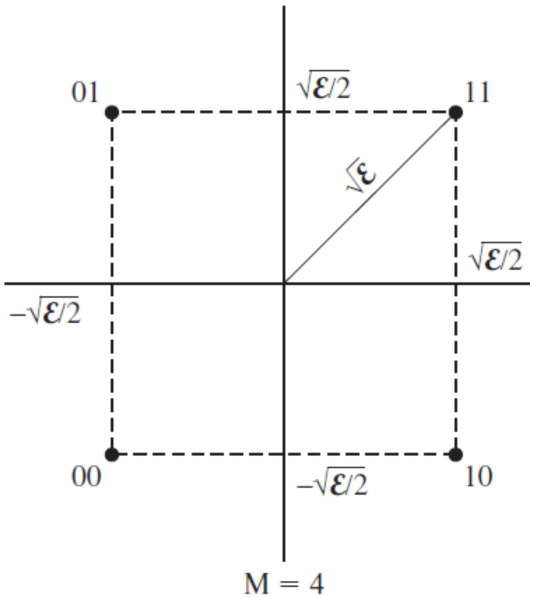
$$= A \cos\left[2\pi \left(f_c + \frac{1}{4T} I_n\right) t - \frac{1}{2} n\pi I_n + \theta_n\right], \qquad nT \le t \le (n+1)T$$

$$f_1 = f_c - \frac{1}{4T}$$

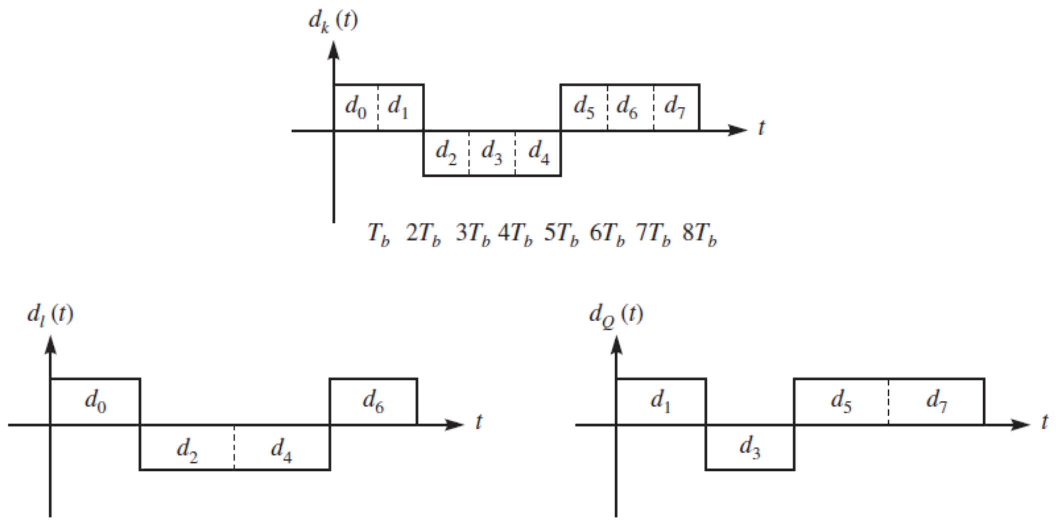
$$f_2 = f_c + \frac{1}{4T}$$

$$s_i(t) = A \cos\left[2\pi f_i t + \theta_n + \frac{1}{2} n\pi (-1)^{i-1}\right], \qquad i = 1, 2$$

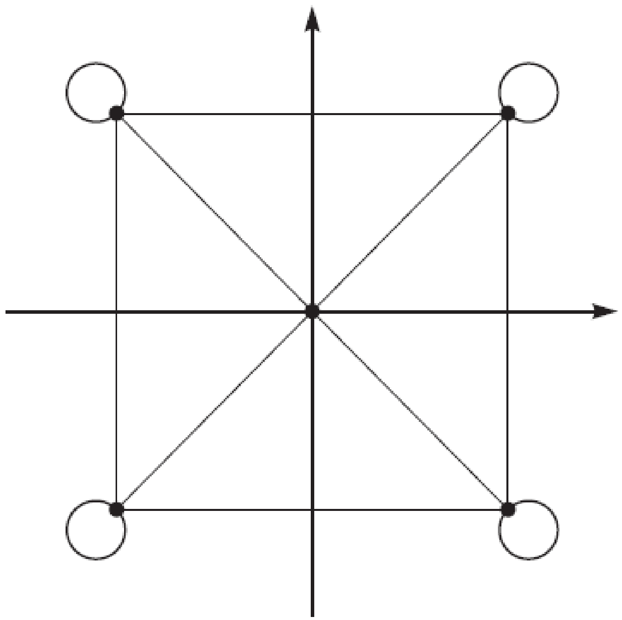
# Offset QPSK (OQPSK)



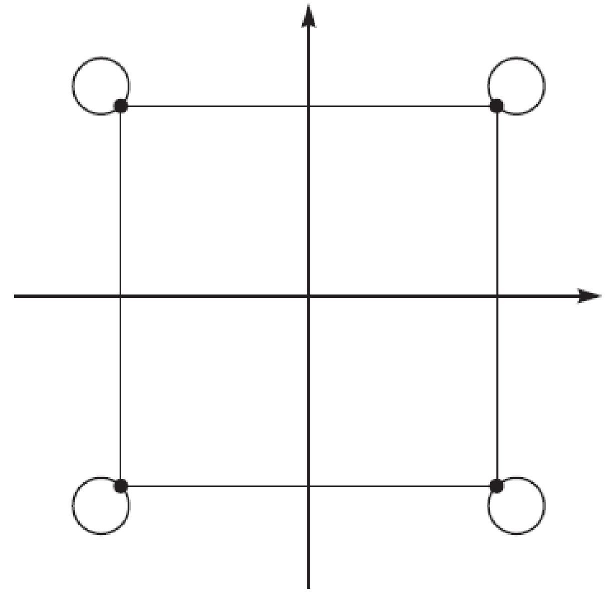

A possible mapping for QPSK




The in-phase and quadrature components for QPSK



Possible phase transitions in QPSK signaling



Possible phase transitions in OQPSK signaling

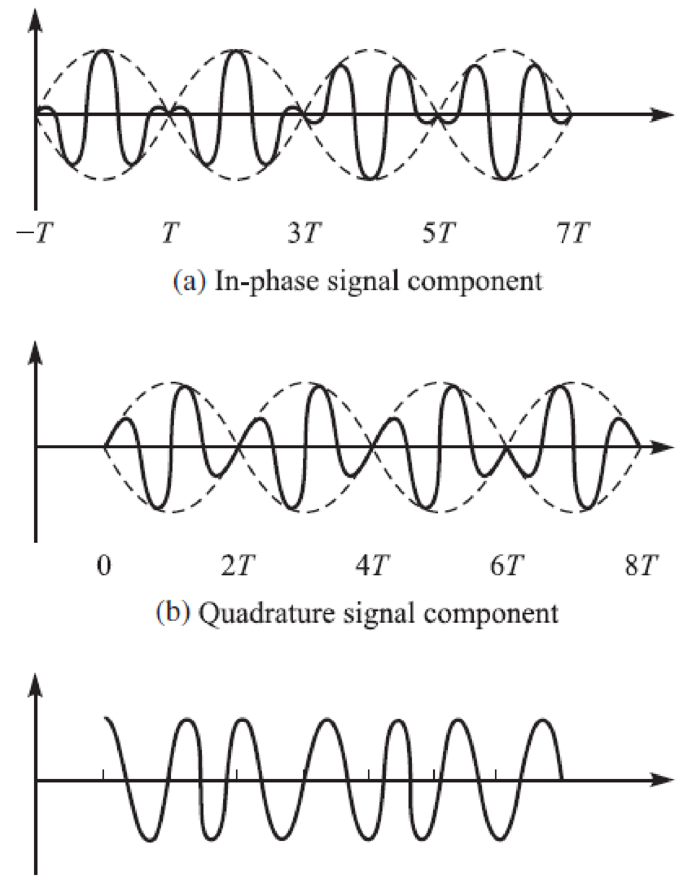
$$s(t) = A\left[\left(\sum_{n=-\infty}^{\infty} I_{2n} g(t - 2nT)\right)\cos 2\pi f_c t + \left(\sum_{n=-\infty}^{\infty} I_{2n+1} g(t - 2nT - T)\right)\sin 2\pi f_c t\right]$$

$$s_l(t) = A\left[\sum_{n=-\infty}^{\infty} I_{2n} g(t - 2nT)\right] - j\left[\sum_{n=-\infty}^{\infty} I_{2n+1} g(t - 2nT - T)\right]$$

$$s_l(t) = A\left[\sum_{n=-\infty}^{\infty} I_{2n} g(t - 2nT)\right] - j\left[\sum_{n=-\infty}^{\infty} I_{2n+1} g(t - 2nT - T)\right]$$

MSK may also be represented as a form of OQPSK. Specifically, we may express the equivalent lowpass digitally modulated MSK signal

$$g(t) = \begin{cases} \sin \frac{\pi t}{2T} & 0 \le t \le 2T \\ 0 & \text{otherwise} \end{cases}$$



(a) In-phase signal component


(b) Quadrature signal component


(c) MSK signal [sum of (a) and (b)]

# Comparing MSK, OQPSK, and QPSK